

CONSTRAINTS ON THE PHOTON MASS WITH FAST RADIO BURSTS

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ABSTRACT

Fast radio bursts (FRBs) are radio bursts characterized by millisecond durations, high Galactic latitude positions, and high dispersion measures. Very recently, the cosmological origin of FRB 150418 has been confirmed by Keane et al. (2016), and FRBs are now strong competitors as cosmological probes. The simple sharp feature of the FRB signal is ideal for them to probe some of the fundamental laws of physics. Here we show that by analyzing the delay time between different frequencies, the FRB data can place stringent upper limits on the rest mass of the photon. For FRB 150418 at $z = 0.492$, one can potentially reach $m_\gamma \leq 5.2 \times 10^{-47}$ g, which is 10^{20} times smaller than the rest mass of electron, and is about 10^3 times smaller than that obtained using other astrophysical sources with the same method.

Subject headings: radio continuum: general — intergalactic medium — plasmas

1. INTRODUCTION

Maxwell's equations, the successful theory of classical electromagnetic mechanical, have a fundamental prediction that all electromagnetic radiation propagate in vacuum at the constant speed c , independent of frequency of the wave. This is also adopted as the second postulate of Einstein's theory of special relativity. As a result, the rest mass of the photon, the fundamental quanta of electromagnetic fields, should be strictly zero.

Testing the correctness of this prediction is one of the most intriguing tasks in modern physics. It is closely related to many fundamental questions such as charge conservation and quantization, the possibility of charged black holes, etc. (Goldhaber & Nieto 1971; Tu et al. 2005), and hence, the need to push its verification as far as possible. According to the uncertainty principle, it is impossible to do any experiment that would firmly establish that the photon rest mass is exactly zero. The ultimate upper limit on the photon rest mass would be $m_\gamma \leq \hbar/(\Delta t)c^2 \approx 10^{-66}$ g, when the age of the Universe ($\sim 10^{10}$ years) is used.

From the theoretical point of view, a nonzero photon mass could be accommodated in a unique way by chang-

ing the inhomogeneous Maxwell's equations to the Proca equations (Proca 1936; Barrow & Burman 1984). Gauge invariance is replaced by the Lorentz gauge, so that a mass term can be added to the Lagrangian density for the electromagnetic field by invoking a characteristic length associated with the photon rest mass, $\mu_\gamma^{-1} = \hbar/m_\gamma c$, to describe the effective range of the electromagnetic interaction (Proca 1936). In this case, the electric and magnetic potentials themselves have a physical significance, not just through their derivatives, which would lead to far-reaching implications. For instance, the speed of light in free space would depend on its frequency, longitudinal electromagnetic waves could exist, magnetic dipole fields would suffer more rapid fall-off with distance due to the addition of a Yukawa component to the magnetic potential, and so on. All of these effects have been employed to derive increasingly restrictive constraints on the photon rest mass, either through laboratory experiments or astrophysical and cosmological observations (Goldhaber & Nieto 1971; Tu et al. 2005; Pani et al. 2012).

The most direct method of constraining the photon mass is to detect a possible frequency dependence of the speed of light. When the laboratory conditions eventually impose a limit, astronomical events afford the best opportunity for obtaining higher precision measurements on the relative speed of electromagnetic radiation at different wavelengths. The first attempt that took advantage of astronomical distances was the comparison of the arrival time of optical and radio emission from flare stars (Lovell et al. 1964), with a constraint on the photon mass of $m_\gamma \leq 1.6 \times 10^{-42}$ g. With a measurement of the dispersion in the arrival time of optical wavelengths of 0.35 and 0.55 μm from the Crab Nebula pulsar, a stringent limit on the possible speed dependence on frequency was set (Warner & Nather 1969), but the corresponding limit on the photon mass was only $m_\gamma \leq 5.2 \times 10^{-41}$ g. Assuming that the radio and gamma-ray photons of gamma-ray bursts (GRBs) have the same origin and that they are emitted at the same time, Schaefer (1999) set the most stringent limit on the frequency dependence of speed of

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light up to now, implying a photon mass $\leq 4.2 \times 10^{-44}$ g, by analysing the arrival time delay between radio and the gamma-ray emission from GRB 980703 at high redshift.

The discovery of Fast Radio Bursts (FRBs) (Lorimer et al. 2007; Thornton et al. 2013) makes it possible to apply these new probes to constrain the photon mass. The pulse arrival times follow the ν^{-2} law, which is consistent with the propagation of radio waves through a cold plasma. The high values of DM (dispersion measure), if contributed from the inter-galactic medium (IGM), would place these bursts in the redshift range of 0.5 to 1 (Thornton et al. 2013). Even though the distance scale and physical origin of these events are still subject to debate, the extra-galactic origin of FRBs is suggested by an increasing number of recent observations (Kulkarni et al. 2014; Kulkarni et al. 2015; Masui et al. 2015; Margalit & Loeb 2015; Saint-Hilaire et al. 2014), especially the detection of ten additional bursts from the direction of FRB 121102 (Spitler et al. 2016), and the potential applications of FRBs as cosmological probes have been suggested (Deng & Zhang 2014; Gao et al. 2014; McQuinn 2014; Zheng et al. 2014; Zhou et al. 2014; Macquart & Johnston 2015). Very recently, Keane et al. (2016) report the first precise localization for an FRB thanks to the identification of a fading radio transient, providing the first redshift measurement to FRB 150418 ($z = 0.492 \pm 0.08$).¹ The simple, sharp, temporal feature of the FRB signal allows one to easily derive the observed time delay between different frequencies. These time lags are usually small (~ 1 s), which make FRBs one of the best candidates for constraining the rest mass of photon.

In this letter, we study the prospects of constraining the photon rest mass with FRBs in detail. Our original motivation is the discovery of a radio wave in FRB 150418 with an approximately 0.815 s delay time that occurs simultaneously from frequencies 1.2 to 1.5 GHz. We will show that FRBs can be a new powerful tool for constraining the photon mass. Throughout this letter, we adopt the cosmological parameters recently derived from nine-year *Wilkinson Microwave Anisotropy Probe* (WMAP) observations (Hinshaw et al. 2013): $\Omega_m = 0.286$, $\Omega_\Lambda = 0.714$, $\Omega_b = 0.046$, and $h_0 = 0.69$.

2. VELOCITY DISPERSION BY THE NON-ZERO MASS OF PHOTON

Assuming that the photon has a non-zero rest mass m_γ , according to Einstein's special relativity, the energy equation can be written as

$$E = h\nu = \sqrt{p^2 c^2 + m_\gamma^2 c^4}. \quad (1)$$

¹ However, Williams & Berger (2016) suggest that the proposed radio transient is a common Active Galactic Nuclei (AGN) variability and unrelated to FRB 150418, and hence that the redshift determination may not be justified. While, it has been proposed that the chance probabilities to have such a highly variable AGN in the Parkes beam and to have the radio source just start to decay after the FRB are low (Li & Zhang 2016). Also the early fading of the radio transient can be understood as an afterglow of a relativistic explosion with energy comparable to a short GRB (Zhang 2016).

In vacuum, the dispersion relation between the speed of photon v and frequency ν is

$$v = \frac{\partial E}{\partial p} = c \sqrt{1 - \frac{m_\gamma^2 c^4}{E^2}} \approx c \left(1 - \frac{1}{2} A \nu^{-2}\right), \quad (2)$$

where $A = m_\gamma^2 c^4 / h^2$. Equation (2) shows that the high energy/frequency photons travel in vacuum faster than the low energy/frequency photons. Two photons emitted simultaneously by a source would arrive on Earth with a time delay if they have different frequencies.

Here we consider two photons (one with higher frequency ν_h and the other with lower frequency ν_l) are emitted simultaneously from a source at redshift z . For the high energy photon, the time of arrival to the Earth is set to be $z = 0$. Since the higher energy photon would arrive earlier than the lower one, the arrival time corresponds to $z = -\Delta z$ ($\Delta z \ll 1$) for the lower one. Taking into account of the cosmological expansion, one can derive the comoving distance from the source to the Earth $x(z, \nu_h) = \frac{c}{H_0} \int_0^z \left[1 - \frac{1}{2} A \nu_h^{-2} \frac{1}{(1+z')^2}\right] \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}}$ for the higher energy photon, and $x(z, \nu_l) = \frac{c}{H_0} \int_{-\Delta z}^z \left[1 - \frac{1}{2} A \nu_l^{-2} \frac{1}{(1+z')^2}\right] \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}}$ for the lower energy photon. Because the comoving distance traveled by these two photons should be the same (i.e., $x(z, \nu_h) = x(z, \nu_l)$), the observer-frame delay time ($\Delta t_{m_\gamma \neq 0}$) between these two photons can be expressed as

$$\Delta t_{m_\gamma} = \frac{\Delta z}{H_0} = \frac{A}{2H_0} (\nu_l^{-2} - \nu_h^{-2}) H_1(z), \quad (3)$$

where $H_0 = 100 h_0 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is the Hubble constant and

$$H_1(z) = \int_0^z \frac{(1+z')^{-2} dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}}. \quad (4)$$

So the photon mass can be constrained as

$$m_\gamma = hc^{-2} \left[\frac{2H_0 \Delta t_{m_\gamma}}{(\nu_l^{-2} - \nu_h^{-2}) H_1(z)} \right]^{1/2}, \quad (5)$$

which can be simplified as

$$m_\gamma = (1.56 \times 10^{-47} \text{ g}) \left[\frac{\Delta t_{m_\gamma}}{(\nu_{l,9}^{-2} - \nu_{h,9}^{-2}) H_1(z)} \right]^{1/2}, \quad (6)$$

where ν_9 is the radio frequency in units of 10^9 Hz.

FRB 150418 was detected by the Parkes radio telescope on 2015 April 18 UTC (Keane et al. 2016). Rapid multi-wavelength follow-up identified a fading radio afterglow, which was used to identify the host galaxy. At the position of the afterglow, imaging on the Subaru and Palomar 200-inch telescope and spectroscopy with FOCAS on Subaru and DEIMOS on Keck reveal the galaxy's redshift to be $z = 0.492$. From the frequency-dependent delay of FRB 150418 (see Figure 1 of Ref. Keane et al. (2016)), we can obtain an approximate $\Delta t = 0.815$ s delay time between frequencies $\nu_l = 1.2$ GHz and $\nu_h = 1.5$ GHz. In principle, the total delay time may contain several contributions (Gao et al. 2015; Wei et al. 2015), including, e.g., an intrinsic (astrophysical) time delay be-

tween two test photons, a time delay caused by effects of Lorentz invariance violation (if this exists), and a time delay from the dispersion process by the line-of-sight free electron content. Assuming that all of the observed time lag can be attributed to a non-zero photon mass, a conservative upper limit for m_γ can be estimated.² As shown in Fig. 1, a strict limit on the photon rest mass from FRB 150418 is $m_\gamma < 5.2 \times 10^{-47}$ g, which is 10^3 times better than that obtained by GRBs (Schaefer 1999). We note that our restriction has been generally confirmed later by Bonetti et al. (2016).

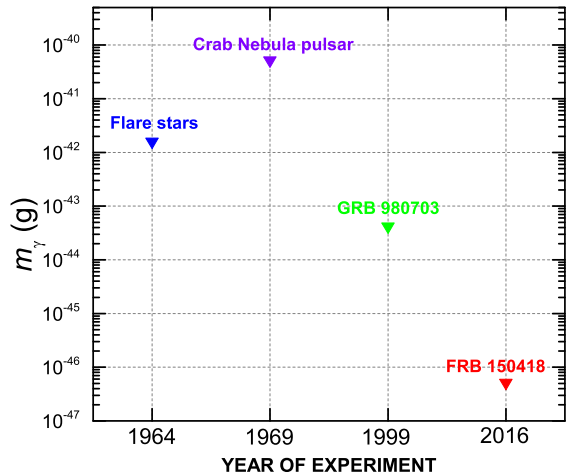


Figure 1. Strict upper limits on the photon rest mass from detection of the frequency dependence of speed of light from astrophysical sources. The upper limits from flare stars, Crab nebula pulsar and GRB 980703 are taken from Lovell et al. (1964), Warner & Nather (1969), and Schaefer (1999), respectively.

3. VELOCITY DISPERSION BY THE PLASMA EFFECT

According to the characteristic observational feature of FRBs, i.e., that the arrival time delay at a given frequency ν follows a ν^{-2} law, the observed time delay Δt between different frequencies should be mainly contributed by the non-zero photon mass effect and the plasma effect via the dispersion process from the line-of-sight free electron content, i.e., $\Delta t \approx \Delta t_{m_\gamma} + \Delta t_{\text{plasma}}$. Note that Δt_{plasma} has the same sign as Δt_{m_γ} , as both predict higher energy photons travel faster than the lower energy photons (see below). In our calculation, we have for simplicity assumed that all of the observed time delay is caused by the non-zero photon mass effect. For completeness, we will also test the case by subtracting the contribution from the plasma effect in Δt . It should be underlined that the measured dispersion has contributions from the Milky Way, intervening intergalactic medium (IGM), and FRB host galaxy. Since the number density of electrons in host galaxy is hard to know and the contribution of the Milky Way dispersion is negligible, we just consider the IGM dispersion.

We assume that the intervening intergalactic medium between the source and the Earth is fully ionized hy-

drogen and singly-ionized Helium when the source located very late after the re-ionization epoch of the Universe (e.g., $z < 3$), and the ionized IGM is homogeneously distributed. The plasma frequency of IGM is $\omega_p = \sqrt{4\pi e^2 n_e / m_e} = 5.64 \times 10^4 n_e^{1/2} \text{ s}^{-1}$. The number density of electrons in IGM increases with the redshift, $n_e = n_e(0)(1+z)^3$. We adopt the mass fraction of Helium in IGM is $Y = 0.24$ (Yoshida et al. 2003), the free electron number per baryon can be estimated to be 0.82. So the local number density of free electrons in IGM is $n_e(0) = 0.82 n_b(0) = 9.23 \times 10^{-6} \Omega_b h_0^2 \text{ cm}^{-3}$ (Inoue 2004), where $n_b(0) = 3\Omega_b H_0^2 / (8\pi G m_p)$ is the local baryon number density and Ω_b is the current baryon fraction of the Universe. The IGM magnetic effect on the velocity dispersion of photons can be neglected, since the Lamour frequency $\omega_L = eB/m_e c = 17.6(B/\mu\text{G}) \text{ s}^{-1}$ is much smaller than ω_p for typical IGM with $n_e > 10^{-6} \text{ cm}^{-3}$ and $B < 10^{-6} \text{ G}$. Due to the plasma effect, the speed of a photon with energy E travelling in IGM at redshift z is

$$v_p(E) = c \sqrt{1 - \left(\frac{E_p(z)}{E(z)} \right)^2} \cong \left[1 - \frac{\nu_p^2(0)(1+z)}{2\nu^2} \right], \quad (7)$$

for $E(z) > E_p(z)$ (i.e., $\nu(z) > \nu_p(z)$), where $\nu_p(z) = (1+z)^{3/2} \nu_p(0)$, $\nu(z) = (1+z)\nu(0)$, and

$$\nu_p^2(0) = \left(\frac{\omega_p(0)}{2\pi} \right)^2 = \frac{0.82 e^2 n_b(0)}{\pi m_e}. \quad (8)$$

This dispersion also leads photons with higher energies travelling faster than those with lower energies (see Equation (7)).

Similar to the derivations of the formulas in Section 2, the time lag caused by the IGM plasma effect can be expressed as

$$\Delta t_{\text{plasma}} = \frac{\Delta z}{H_0} = \frac{\nu_p^2(0)}{2H_0} (\nu_l^{-2} - \nu_h^{-2}) H_2(z), \quad (9)$$

where

$$H_2(z) = \int_0^z \frac{(1+z') dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}}. \quad (10)$$

Equation (9) can be simplified as

$$\Delta t_{\text{plasma}} \approx 3.64 \text{ s} \left(\frac{\Omega_b h_0}{0.032} \right) H_2(z) (\nu_{l,9}^{-2} - \nu_{h,9}^{-2}). \quad (11)$$

With the redshift of FRB 150418, an estimation on Δt_{plasma} between frequencies $\nu_l = 1.2 \text{ GHz}$ and $\nu_h = 1.5 \text{ GHz}$ from Equation (11) is $\Delta t_{\text{plasma}} = 0.491 \text{ s}$. Now the stricter upper limit on the photon mass comes to be $m_\gamma \leq 3.3 \times 10^{-47} \text{ g}$ for the case of $\Delta t_{m_\gamma} = \Delta t - \Delta t_{\text{plasma}} = 0.324 \text{ s}$. If we have a better understanding of the host galaxy dispersion, our constraint would be further improved in some degree.

4. SUMMARY AND DISCUSSION

In this letter, we have shown that FRBs can be used to place severe limits on the photon mass m_γ . From the frequency-dependent delay of FRB 150418, we obtained approximately a delay time (Δt) = 0.815 s of the radio pulse at $\nu_l = 1.2 \text{ GHz}$ relative to that at $\nu_h = 1.5 \text{ GHz}$.

² With the similar approach, i.e., based on knowledge of the FRB redshift and assuming the time delay is dominated by the gravitational field of the Milky Way, Tingay & Kaplan (2016) set a strict limit on the Einstein Equivalence Principle from FRB 150418.

Considering the delay time was caused by the non-zero photon mass ($m_\gamma \neq 0$) effect and adopting the possible redshift $z = 0.492$ for FRB 150418, the severe limit on the photon mass is $m_\gamma \leq 5.2 \times 10^{-47}$ g, which represents an improvement of three orders of magnitude over the results by GRBs from Schaefer (1999).

Notice that this is a conservative upper limits: the inclusion of contributions from the other neglected potential contributions to Δt can make this limit even more stringent. If the time delay between different frequencies is mainly contributed by the plasma effect, more severe constraints would be achieved (e.g., $m_\gamma \leq 3.3 \times 10^{-47}$ g) if the contribution from the plasma effect has been subtracted in Δt (i.e., $\Delta t_{m_\gamma} = \Delta t - \Delta t_{\text{plasma}}$). Moreover, Keane et al. (2016) found that the measurement of the cosmic density of ionized baryons Ω_{IGM} from the dispersion measure and the redshift of FRB 150418 is in agreement with the expectation from WMAP observations, which leads one to conclude that the observed time delay for FRB 150418 is highly dominated by plasma dispersion, with very small contribution possibly from the non-zero photon mass effect. Optimistically, if the non-zero photon mass effect is responsible for 10.0% of Δt of FRB 150418, the upper limit on m_γ could be set to $m_\gamma < 1.6 \times 10^{-47}$ g, which is closer to the current most stringent constraints as adopted by the Particle Data Group (Amsler et al. 2008). The results presented here show the potentially high benefits to be obtained when more FRBs are observed, especially if the redshifts of FRBs can be more likely to be measured in the future.

While there may be multiple physical origins for the population of FRBs, the extra-galactic origin of at least some FRBs is receiving increasingly supports from the observational data (e.g., Spitler et al. 2016). We find that even if FRB 150418 originate within our Local Group (1 Mpc; Karachentsev & Kashibadze 2006) or the Local Supercluster (50 Mpc; Tully et al. 2014), a strict limit on the photon mass of $m_\gamma < 1.9 \times 10^{-45}$ g or $m_\gamma < 2.7 \times 10^{-46}$ g can be still achieved, which is already 10 times or 100 times smaller than that obtained by GRBs using the same method (Schaefer 1999).

Currently, the detection rate of FRBs is relatively low, mainly due to the lack of either necessary high time resolution or a wide field of view in any of the current telescopes. Future radio transient surveys such as the Square Kilometer Array would break through these obstacles (Dewdney et al. 2009), and are expected to discover and precisely localize an increasing number of FRBs. With the abundant observational information in the future, the mysteries of the physical nature of FRBs are expected to be eventually unveiled, and their capability for testing fundamental physics, as discussed here, will find increased use.

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